Chapter 4

- Purpose:
  - Summation notation
  - Random variable
  - Expected Value
  - Project Section

Section 4-1: Summation Notation

- The symbol for “add a bunch of numbers together” is $\Sigma$. This is the capital letter “sigma” in the Greek alphabet.
- Instead of writing $2+4+6+...+100 = \Sigma 2i$
  This says, “The sum of $2i$, as $i$ increases from 1 to 50,” which means $2*1+2*2+2*3+...+2*50$
- Example 1, 2, 3 (p. 65-66)...

Section 4-3: Using Excel to Compute Sums

- $\Sigma i^2 + 6i$, from 1 to 100.
- If you compute by hand, it would take a while.
- Or you can use Excel, p. 68.

Section 4-4: Random Variables

- A random variable is a variable whose value is a numerical outcome of a random variable.
Section 4-5: Expected Value

- Expected Value is the predicted average of all the outcomes, in other words the weighted average of a random variable.

\[ E(X) = \sum x \cdot p \]

- \( x \) is the random variable
- \( p \) is the probability of each outcome occurring

Expected Value Sample

- You pay $2 to play a dice game where you roll 2 dice. If the sum is 7, you win $5. If the sum is 2, you win $10. Calculate the expected value?

- If you roll a 7, you win $5, but pay a cost of 2, thus 
  \[ 5 - 2 = 3 \] is the return

- If you roll a 2, you win $10, but also pay a cost of 2, thus 
  \[ 10 - 2 = 8 \] is the return

- If you roll anything else besides 7 or 2, you lose $2

Expected Value Sample (cont.)

- Thus, for every bet, you will lose about 0.89 cents on the $2 bet

<table>
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<th>x</th>
<th>p</th>
<th>x*p</th>
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<tr>
<td>3</td>
<td>1/6</td>
<td>1/2</td>
</tr>
<tr>
<td>8</td>
<td>1/36</td>
<td>2/9</td>
</tr>
<tr>
<td>-2</td>
<td>29/36</td>
<td>-1 11/18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8/9</td>
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</tbody>
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Section 4-6: Project

- How can expected value help us with the decision on whether or not to attempt a loan work out?
Chapter 5

- Purpose:
  - Conditional Probability
  - Independence
  - Project

Section 5-1: Conditional Probability

- **Conditional Probability** is the probability of one event depending on another.

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} \]

- This is the probability of A and B happening at the same time, divided by the probability of B happening.
- In Bus210, this is called the general rule of multiplication.

Conditional Probability, Example 3

- Suppose 40% of the population likes country music, and 30% likes gospel, and 10% like both. What is the probability that someone likes country given they like gospel?

\[ P(G|C) = \frac{P(G \cap C)}{P(C)} \]
\[ P(G|C) = \frac{0.10}{0.40} \]
\[ P(G|C) = 0.25 \]

- Will you get the same answer from \( P(C|G) \)?
- No, but why?

P(G|C) and P(C|G), same answer?

- Suppose 40% of the population likes country music, and 30% likes gospel, and 10% like both. What is the probability that someone likes country given they like gospel?

\[ P(C|G) = \frac{P(C \cap G)}{P(G)} \]
\[ P(C|G) = \frac{0.10}{0.30} \]
\[ P(C|G) = 0.3333 \]

\[ P(G|C) = 0.25 \]
Section 5-2: Independence

- Two events are called independent if and only if the outcome of one event does not affect the outcome of the other.
- Good example: if you flip a coin and get tail, this will have no affect on the next flip of a coin.
- If independent, then
  \[ P(A \cap B) = P(A) \times P(B) \]

Non-Independence Example 2

- Suppose there is a 30% probability that the temperature falls below freezing. If we know this happens, then the probability of snow is 50%. What is the probability of snow AND the temperature falling below freezing?
  - “If we know this happens,” meaning “given” freezing.
  - “Snow AND the temperature falling,” meaning \( P(S \cap F) \).
  - Thus...
    \[ P(S \mid F) = \frac{P(S \cap F)}{P(F)} \text{ rearrange...} \]
    \[ P(S \cap F) = P(S \mid F) \times P(F) \]
    \[ P(S \cap F) = 0.50 \times 0.30 = 0.15 \]

Section 5-3: Testing for Independence

- Example 1: First calculate \( P(A) \) and \( P(B) \) and multiply them to get \( P(A \cap B) \)
- If independent, then there should only 3 ways out of 36 that have seven and a red die.
- If you roll two die, one red and one green, at the same time. Only 3 combination can include a red AND an even number.
  - (red = 2, green = 5), (red = 4, green = 3), and (red = 6, green = 1)
- Yes, this is independent!

Section 5-4: Conditional and Independent

- If events A and B are independent, and Z is an event (not necessarily independent) then:
  \[ P(A \cap B \mid Z) = P(A \mid Z) \times P(B \mid Z) \]
- This rule can apply to any number of independent events. For example,
  \[ P(A \cap B \cap C \mid Z) = P(A \mid Z) \times P(B \mid Z) \times P(C \mid Z) \]
Conditional and Independent Example 1

- If Ralph catches a cold, there is a 15% chance that he will pass the cold to his friend Terrance, and a 35% chance that he will pass the cold to his friend Joseph. Assuming that Terrance and Joseph don’t have any contact with each other, what is the probability that both Terrance and Joseph will get sick?

\[
P(T \cap J | R) = P(T | R) \times P(J | R) \\
P(T \cap J | R) = 0.15 \times 0.30 = 0.0525
\]

Section 5-5: Project Work

- How can probability help us with the decision on whether or not to attempt a loan work out?