Tutorial: Newton's law of gravitation

Take $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$, Earth mass = $5.98 \times 10^{24} \text{ kg}$, Earth radius = $6.37 \times 10^6 \text{ m}$.

1) A satellite of mass $m$ is located a distance $r$ from the center of the Earth of mass $M$

![](Image)

a. What is the force acting on the satellite by the Earth (direction, magnitude, units)? Use symbols not numbers.

b. How does the force acting on the Earth by the satellite compare to the force found in a)? Explain.

c. As we all know, the weight of a mass $m$ is given by $W_{me} = mg$. What is the acceleration of gravity $g$ at the location of the satellite (i.e. at a distance $r$ from the center of the Earth)? Use symbols not numbers.
d. The international space station is at an average altitude of 370 km above the surface of the Earth. What is the weight of an astronaut in the space station if her mass is 65 kg? How does it compare with her weight on the surface of the Earth? (Earth radius = 6370 km.)

e. Assume that the satellite is in uniform circular motion around the Earth.

i. Apply $F_{net} = m \ a$ to find the period of revolution $T$ of the satellite. (This is Kepler's 3\textsuperscript{rd} law). Use symbols not numbers.

ii. For what distance $r$ would $T$ be equal to 24 hours? If the satellite moves in the plane of the equator in the same direction as the Earth rotates about its axis, the satellite is always at the vertical of the same point on Earth. This is what is called a geosynchronous satellite.
iii. How fast should you throw horizontally a baseball at the equator so that it orbits the Earth? Does it matter whether you throw it north, south, east or west?

iv. If the Earth could be moved to the distance of Jupiter and placed on a circular orbit around the Sun, what would be the length of the year on Earth? Jupiter is about 5.2 times as far from the Sun as Earth is.
2) Give the definition of the work done by a force $\vec{F}$ acting over a displacement $\Delta \vec{r}$ (draw a picture and give the formula).

3) How would you modify the definition given in 2) if the force changes in magnitude and/or direction over of the displacement, or if the displacement is not along a straight line?

4) The satellite of question 1) is moved from a distance $r = r_i$ to and a distance $r = r_f > r_i$ along a radial line. How much work is done by the gravitational force from $r = r_i$ to $r = r_f$? Use symbols not numbers.
5) Is the gravitational force conservative? Explain.

6) What is the potential energy associated with the Earth satellite system? Is the mechanical energy of the system Earth satellite conserved? Use symbols not numbers.

7) At some instant, the satellite is at position $r = r_1$ with velocity $v = v_1$. At a later instant, it is at position $r = r_2$ with velocity $v = v_2$. Apply the conservation of mechanical energy to relate $r_1$ and $v_1$ to $r_2$ and $v_2$. Use symbols not numbers.
The relationship found in 7) always applies for any motion due to gravitational interaction. Use it in questions 8 and 9.

8) An object is fired vertically from the North Pole with a velocity $v$. What is the maximum altitude $h$ attained by the object ($r = R + h$)? Does the object always fall back on the Earth? Find the minimum velocity at which the object escapes the gravitational attraction of the Earth.
9) More challenging! You will need the relationship found in 7). However, you will also need another relationship that we introduced when studying collisions in Phys 221.

Two stars A and B initially at rest and very far from each other crash into one another along a straight line. The radii and masses of A and B are \( R_A = 7 \times 10^8 \) m, \( R_B = 9 \times 10^8 \) m, \( m_A = 10^{30} \) kg, and \( m_B = 2 \times 10^{30} \) kg. What are the velocities of A and B when they collide?