CONSERVATION OF MOMENTUM IN ONE DIMENSION

1. Analyzing collisions

Two experiments are conducted with gliders on a level, frictionless track.

In the first experiment, glider A is launched toward a stationary glider, glider M. After the collision, glider A has reversed direction.

In the second experiment, glider M is replaced by glider N, which has the same mass as glider M. Glider A has the same initial velocity as in experiment 1. After the collision, glider A is at rest.

The mass of glider A is one-fifth that of glider M and glider N (i.e., $m_A = 5m_1$, $m_M = m_N$).

<table>
<thead>
<tr>
<th>Experiment 1</th>
<th>Before collision</th>
<th>After collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\vec{v}_{1A}$</td>
<td>$\vec{v}_{1A}$</td>
</tr>
<tr>
<td>M</td>
<td>$\vec{v}_{1M} = 0$</td>
<td>$\vec{v}_{2M}$</td>
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</tbody>
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<table>
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<tr>
<th>Experiment 2</th>
<th>Before collision</th>
<th>After collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\vec{v}_{1A}$</td>
<td>$\vec{v}_{2A}$</td>
</tr>
<tr>
<td>N</td>
<td>$\vec{v}_{1N} = 0$</td>
<td>$\vec{v}_{2N}$</td>
</tr>
</tbody>
</table>

A. What differences between gliders M and N could account for their differences in behavior during the collisions?

The collision between M and A is more elastic. M and N are made of different materials (e.g., steel for M, clay for N).

B. For experiment 1, draw and label separate force-body diagrams for glider A and glider M at an instant during the collision (i.e., while the gliders are in contact).

1. While the gliders are in contact, how does the net force on glider A compare to the net force on glider M? Discuss both magnitude and direction.

$$\sum F_{net} \text{ on } A = F_{AH} = 3rd \text{ law} \quad F_{HA} = -F_{net} \text{ on } M \quad \text{opposite in direction}$$

How, if at all, would this comparison differ if you had chosen a different instant (while the gliders are still in contact)? Explain.

Same conclusion: the 3rd law applies throughout the collision (of course, $F_{HA}$ and $F_{AH}$ change in magnitude but $F_{HA} = F_{AH}$ always).

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2. Consider the small time interval \( \Delta t \) while the gliders are still in contact. For the two gliders, how does the product \( F_{net} \Delta t \) compare to the product \( F_{nets} \Delta t \)? Discuss both magnitude and direction. Explain.

**Same \( \Delta t \) so from 1)**

Apply Newton's second law \( \vec{F}_{net} = m \frac{\Delta \vec{v}}{\Delta t} \) to each of the colliding gliders in Experiment 1 to compare the change in momentum \( \Delta \vec{p} = m \Delta \vec{v} \) of gliders A and M during the collision. Discuss both magnitude and direction. Explain.

\[
\Delta \vec{p} = m \Delta \vec{v} = \vec{F}_{net} \Delta t
\]

**Using the previous result**: \( \Delta \vec{p}_A = -\Delta \vec{p}_M \)

C. In the spaces provided, draw and label vectors to represent the initial momentum, the final momentum, and the change in momentum of glider A in each experiment.

1. Is the magnitude of the change in momentum of glider A in experiment 1 greater than, less than, or equal to the magnitude of the change in momentum of glider A in experiment 2? Explain.

\( \left| \Delta \vec{P}_{A \text{in}1} \right| > \left| \Delta \vec{P}_{A \text{in}2} \right| \) (see boxes)

2. Is the magnitude of the change in momentum of glider M in experiment 1 greater than, less than, or equal to the magnitude of the change in momentum of glider N in experiment 2? Explain.

Since \( \Delta \vec{P}_{A \text{in}1} = -\Delta \vec{P}_M \)

\( \Delta \vec{P}_{A \text{in}2} = -\Delta \vec{P}_N \)

\( \left| \Delta \vec{P}_M \right| > \left| \Delta \vec{P}_N \right| \)

After the collisions, is the speed of glider M greater than, less than, or equal to the speed of glider N? Explain.

For M and N: \( \vec{p}_1 = 0 \Rightarrow \vec{p}_f = \Delta \vec{p} \)

Thus \( \left| \vec{P}_f \right| > \left| \vec{P}_N \right| \Rightarrow v_f > v_N \)

(use \( \vec{p} = m \vec{v} \) and \( m_M = m_N \))

D. A student compares the final speeds of gliders M and N.

"In experiment 2, glider A transfers all of its momentum to glider N, whereas in experiment 1, glider A still has some momentum left, so glider M does not get as much. Therefore, glider N has a greater final speed than glider M."

Do you agree or disagree with this statement? Explain.

\( \Delta \vec{p}_A \) has greater magnitude in 1 than in 2 because of the change in direction.

Discuss your answers with a tutorial instructor.
II. Applying momentum conservation to systems of multiple objects

An experiment is conducted on a frictionless air track in which a glider, glider C, is launched toward a second glider, glider D.

A. Suppose that glider D is free to move and glider C rebounds.

1. In the spaces provided, draw separate free-body diagrams for each glider and for the system of the two gliders, system S, at an instant during the collision.

   Free-body diagram for glider C
   \[ \begin{align*}
   N_{CT} & \rightarrow \text{C} \\
   F_{CD} & \rightarrow \text{D} \\
   W_{CE} & \rightarrow \text{C}
   \end{align*} \]

   Free-body diagram for glider D
   \[ \begin{align*}
   N_{DT} & \rightarrow \text{D} \\
   F_{DC} & \rightarrow \text{C} \\
   W_{DE} & \rightarrow \text{D}
   \end{align*} \]

   Free-body diagram for system S
   \[ \begin{align*}
   N_{ST} & \rightarrow \text{S} \\
   & \rightarrow \text{S}
   \end{align*} \]

Which forces in your free-body diagrams for glider C and glider D do not have corresponding forces in the free-body diagram for system S?

\[ \overrightarrow{F_{CD}} \quad \text{and} \quad \overrightarrow{F_{DC}}: \text{they are internal forces for system S.} \]

2. The momentum of a system containing multiple objects can be defined to be the sum of the momenta of the constituent objects.

Use this definition to write an expression for the change in momentum of system S in terms of the change in momentum of glider C and of glider D.

\[ \overrightarrow{\Delta P_s} = \overrightarrow{\Delta P_C} + \overrightarrow{\Delta P_D} \]

3. Does the momentum of each of the following change during the collision? Explain how you can tell:

   - glider C: \[ \overrightarrow{F_{netC}} = \overrightarrow{F_{CD}} \neq 0 \Rightarrow \overrightarrow{\Delta P_C} \neq 0 \]
   - glider D: \[ \overrightarrow{F_{netD}} = \overrightarrow{F_{DC}} \neq 0 \Rightarrow \overrightarrow{\Delta P_D} \neq 0 \]
   - system S: \[ \overrightarrow{F_{netS}} = 0 \Rightarrow \overrightarrow{\Delta P_s} = 0. \]

Are your answers consistent with your free-body diagrams and the direction of the net force in each case? If not, resolve any inconsistencies.

4. How, if at all, would your answer about the change in momentum of system S differ if glider D were replaced by a much more massive glider? Explain.

\[ \text{No change:} \quad \overrightarrow{\Delta P_s} = 0 \quad \text{since} \quad \overrightarrow{F_{netS}} \text{would still be 0.} \]
B. A second experiment is performed in which glider D is fixed in place. Glider C is launched toward glider D with the same velocity as in the first experiment, and it rebounds with the same speed that it had initially.

1. In the spaces provided, draw separate free-body diagrams for each glider and for the system of the two gliders at an instant during the collision in this second experiment.

![Free-body diagrams](image)

Explain how the fact that glider D is fixed in place is reflected in your free-body diagrams.

\[ \vec{F}_{\text{net},D} = 0 \]

2. Does the momentum of each of the following change during the collision? Explain.
   - glider C  \[ \vec{F}_{\text{net},C} \neq 0 \quad \Delta \vec{p}_C \neq 0 \]
   - glider D  \[ \vec{F}_{\text{net},D} = 0 \quad \Delta \vec{p}_D = 0 \]
   - system S  \[ \vec{F}_{\text{net},S} \neq 0 \quad \Delta \vec{p}_S \neq 0 \]

C. Consider the two experiments described above. When the momentum of an object or system of objects did not change:
   - were external forces exerted on the object or system? **Yes:** (e.g. \( \vec{W}_{\text{ext}} \))
   - was there a net force on the object or system? **No:** \( \vec{F}_{\text{net}} = 0 \Rightarrow \Delta \vec{p} = 0 \)

D. When the momentum of an object or system of objects does not change with time, the momentum of the object or system is said to be conserved.

On the basis of your results above, describe the circumstances under which the momentum of an object or system of objects is conserved.

Since \( \vec{F}_{\text{net}} \), \( \Delta \vec{p} = \vec{F}_{\text{net}} \), if \( \vec{F}_{\text{net}} = 0 \), \( \Delta \vec{p} = 0 \).

E. Two students discuss the second experiment, in which glider D is fixed in place.

Student 1: "When one object hits another, the momentum of the system is always conserved."

Student 2: "That's right, the momentum of glider C is the same before and after the collision."

Describe the error in each student's statement.

**Student 1:** in B) \( \Delta \vec{p}_S \neq 0 \).

**Student 2:** \( \vec{p}_D = \vec{p}_C \) same magnitude, but \( \neq \) direction