Two’s complement

-18 in 10's complement
- Start with the positive number (3 digits) 018
- Write the 9's complement (0→9, 1→8, etc.) 981
- To get the 10's complement, add 1
- 982
- Same as doing 1000 – 18
- With 3 digits, 900 to 999 is -100 to -1
- 000 to 099 is 0 to 99

What about 0?
- Using 3 digits
- +0 is 000
- -0?
  - 000 (+0)
  - 999 (9's complement)
  - 999 + 1 = 000 + carry = 1(10's complement)
  - Always ignore the carry, so
  - -0 is 000
- Same as +0

Rules
- Positive numbers start with 0
- Negative numbers start with 9
- Apply the usual rules of arithmetic
e.g. with 3 digits:
  - 25 – 30 = 025 + 970 = 995
  - 995 is the usual -5 (995 = 1000 – 5)
  - -10 -15 = 990 + 985 = 2975 (ignore the carry!). 975 is the usual -25
- Overflow if the carry going into the sign digit (0 or 9) is not equal to the carry coming out of it. Remedy → use more digits.

10's complement
- How to represent negative numbers?
  - Use a sign → but -0 is the same as +0
  - 10’s complement
- Example
  - pick a number of digits (sign + magnitude): e.g. 3
  - positive numbers: +18 → 018
  - negative numbers: −18 → 982
- Why?

2’s complement
- Positive numbers start with 0
e.g with 4 digits
  - 7 is 4 + 2 + 1 = 111₂ = 0111₂
- Negative numbers?
  - Start with the positive value: +7 → 0111₂
  - 1’s complement: 1000
  - 2's complement (add 1): 1001 (= -8 + 1)
- Positive numbers start with 0, negative numbers start with 1
Examples

- **Rules:**
  1. $1 + 1 = 10$ (0, carry =1)
  2. $0 + 1 = 1$, $1 + 0 = 1$, $0 + 0 = 0$
  3. Ignore any carry out of the sign bit (overflow?)

- **37 + 19 (with 8 digits) = 56**
  - 37 = 0 0 1 0 0 1 0 1
  - 19 = 0 0 0 0 1 0 1 1
  - 56 = 0 0 1 1 1 0 0 0

- **-37 + 19 (with 8 digits) = -18**
  - -37 = 1 1 1 1 1 0 1 1
  - 19 = 0 0 0 0 1 0 1 1
  - -18 = 1 1 1 1 1 0 1 1

Overflow

- **100 + 50 (with 8 digits) = 150 (too big!)**
  - 100 = 0 1 1 0 0 0 0 0
  - 50 = 0 0 1 1 0 0 0 0
  - overflow = 1 0 0 1 0 0 0 0

The carry into the sign bit is +1 ≠ carry out of the sign bit is 0

- Fix: use 9 digits
  - 100 = 0 1 1 0 0 0 0 0 0
  - 50 = 0 0 1 1 0 0 0 0 0
  - 150 = 0 1 1 0 0 0 0 0 0

What about multiplication?

- Works as usual if the number of digits is enough to accommodate the answer
- With 3 digits: $-3 = 101$, $-4 = 100$
- However, $-4 \times -3 = 12 = 01100$, doesn't fit into 3 digits
- Fix: sign extend -3 and -4 to as many bits as necessary
- How many? Safe approach: here, double the number of digits. In our example, switch to 6 digits
- $-4 = 111100$, $-3 = 111101$
- Do the multiplication and keep only the last 6 digits for all operations = 001100

Rules (1)

- For any number, can always remove all 0's or all 1's on the left except the last 0 and 1:
  - $000110 = 0110 = +6$
  - $111011 = 1011 = -5$
- To convert to decimal form
  - If positive: $01101 = 2^3 + 2^2 + 2^0 = 13$
  - If negative: $10111$
    - $-2^4 + 2^3 + 2^1 + 2^0 = -9$
  - or convert to positive:
    - 1's complement = 01000
    - 2's complement = 01000 + 1 = 01001
    - and $01001 = 2^3 + 2^2 = 9$
    - so $10111 = -9$

Rules (2)

- Addition: $a + b$
  - Pad a and b on the left to max( number of digits of a, number of digits of b) + 1 — never any overflow, always ignore any outgoing carry on the leftmost digits.
  - $-5 = 1011$ (4 digits)
    - $-3 = 101$ (3 digits)
    - pad to max(4,3) + 1 = 5
    - $-5 + -3 = 11011 + 11101$
    - $1$ $1$ $1$ $0$ $1$ $1$
    - $+ 1$ $1$ $1$ $0$ $1$
    - $---------------$
    - $1$ $1$ $0$ $0$ $0$ $0$ (the outgoing carry = 1 on the leftmost digits is ignored)
    - $11000 = 1000 = -8$

Rules (3)

- Multiplication: $a \times b$
  - Pad a and b on the left to $m$ = number of digits of a + number digits of b; never any overflow, always ignore any outgoing carry on the leftmost digits, for all steps work with just $m$ digits
  - $-5 = 1011$ (4 digits)
    - $-3 = 101$ (3 digits)
    - work with $4 + 3 = 7$ digits for all operations
    - $1$ $1$ $0$ $0$ $1$
    - $+ 1$ $1$ $0$ $1$
    - $---------------$
    - $1$ $1$ $0$ $0$ $1$
    - $+ 1$ $1$ $0$ $1$
    - $---------------$
    - $1$ $1$ $0$ $0$ $1$
    - $+ 1$ $0$ $0$ $1$
    - $---------------$
    - $1$ $0$ $0$ $1$ $1$
    - $+ 1$ $0$ $0$ $1$
    - $---------------$
    - $1$ $0$ $0$ $1$ $1$