Approximate Geometric Mean

Sometimes when you are trying to estimate a quantity it is easier to give lower and upper bounds for that quantity. For example, you may not know how many sandwiches are eaten on the average by a person in a month, but it might be reasonable to assume that this number is somewhere between 1 and 30. In this situation, we could try to approximate the number by taking the average of the upper and lower bounds (so we would use \((1 + 30)/2 = 15.5\) sandwiches per month) but this would give us a number that is over 15 times bigger than the lower bound and only around 2 times smaller than the upper bound.

To get a better number between the lower and upper bounds, we instead use the geometric mean. The geometric mean of numbers \(a\) and \(b\) (where \(a\) is less than \(b\)) is the square root of their product (that is, \(\sqrt{ab}\)). This number is as many times bigger than \(a\) as it is smaller than \(b\). For example, the geometric mean of 4 and 9 is the square root of \(4 \times 9 = 36\), which is 6. Note that 6 is one and half times bigger than 4, and that 9 is also one and a half times bigger than 6. (If instead we had taken the average of 4 and 9 we would get 6.5).

Calculating square roots can be difficult, so sometimes we approximate the geometric mean as follows: If we have two numbers in scientific notation \(- a \times 10^p\) and \(b \times 10^q\) – we take the average of \(a\) and \(b\) and the average of the exponents \(p\) and \(q\). For example, the approximate geometric mean of \(2 \times 10^{15}\) and \(6 \times 10^3\) is \(4 \times 10^9\) (since 4 is the average of 2 and 6 and 9 is the average of 15 and 3). If the sum of the exponents is an odd number, the average wouldn’t be a whole number, so to make this easier to work with we decrease the sum by one (to make it an even number) and multiply the final answer by 3. For example, the approximate geometric mean of 1 and 1000 (that is, of \(10^0\) and \(10^3\)) is 30. (So 30 is 30 times bigger than 1 and 1000 is about 30 times bigger than 30.)