Homework Assignment #2

Due: April 8 (Tuesday)

This assignment will introduce the use of the Cartesian Coordinate System, vector notation, and properties of vectors to find corresponding unit vectors. In addition, the dot/cross product will also be introduced.

All vectors are starting from the origin, O(0,0), and directing to each point, e.g., \( \vec{A} \triangleq \overrightarrow{OA} = 4\hat{i} + \hat{j} \) where \( \hat{i} \) and \( \hat{j} \) are unit vector of \( x \) and \( y \) direction respectively.

1. Express all vectors \( \vec{B}, \vec{C}, \vec{D}, \vec{E}, \vec{F}, \) and \( \vec{G} \) with unit vectors \( \hat{i} \) and \( \hat{j} \).

2. Compute (Addition & Substraction)

   - \( \vec{A} + \vec{D} \)
   - \( \vec{E} + \vec{G} \)
   - \( \vec{C} - \vec{E} \)
   - \( \vec{B} - \vec{E} \)
3. Compute (dot product)

- $\vec{A} \cdot \vec{F}$
- $\vec{E} \cdot \vec{G}$
- $\vec{D} \cdot (\vec{E} + \vec{G})$
- $\vec{B} \cdot \vec{E}$
- $\vec{A} \cdot (-\vec{A})$

4. Compute (cross product)

- $\vec{A} \times \vec{F}$
- $\vec{E} \times \vec{G}$
- $\vec{D} \times (\vec{E} + \vec{G})$
- $\vec{B} \times \vec{E}$
- $\vec{A} \times (-\vec{A})$

5. Analyze your 3. ∼ 4. answers.

*hint:* Recall the definitions of dot/cross products and angles.

6. Compare the forces $F$ required to just start the 900 N lawn roller over a 75 mm step when (a) the roller is pushed and (b) the roller is pulled.
7. A circular cylinder $A$ rests on top of two half-circular cylinder $B$ and $C$, all having the same radius $r$. The weight of $A$ is $W$ and that of $B$ and $C$ is $1/2W$ each. Assume that the coefficient of friction between the flat surfaces of the half-cylinders and the horizontal table top is $f$. Determine the maximum distance $d$ between the centers of the half-cylinders to maintain equilibrium.

![Diagram of cylinders](image)

8. A vector $\vec{F} = F_x \hat{i} + F_y \hat{j} = F_a \hat{a} + F_b \hat{b}$, where $\hat{i}$, $\hat{j}$ and $\hat{a}$, $\hat{b}$ are pairs of perpendicular unit vector in a plane. Show that

$$F_a = F_x \cos \theta + F_y \sin \theta$$
$$F_b = -F_x \sin \theta + F_y \cos \theta$$

and

$$F_x = F_a \cos \theta - F_b \sin \theta$$
$$F_y = F_a \sin \theta + F_b \cos \theta$$

These equations may also be written in the matrix form

$$\begin{bmatrix} F_a \\ F_b \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix},$$

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} F_a \\ F_b \end{bmatrix}.$$