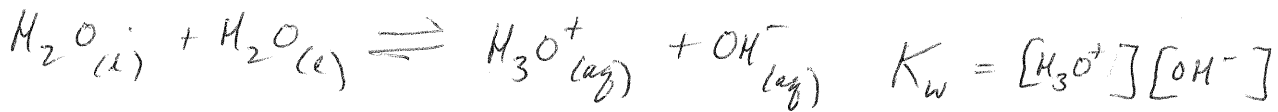
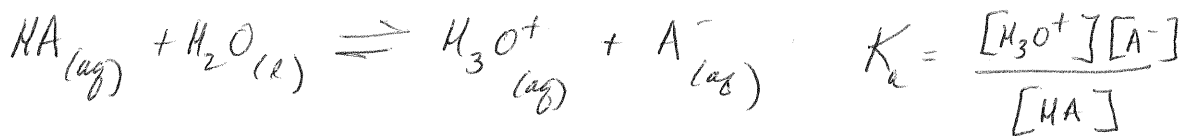


General Solution for Weak Acid pH - Derivation



mass balance: $[HA]_0 = [HA] + [A^-]$

charge balance: $[H_3O^+] = [A^-] + [OH^-]$ read this as the concentration of positive charge = the concentration of negative charge

4 equations 4 unknowns ($[H_3O^+]$, $[OH^-]$, $[A^-]$, $[HA]$)
(or as Don Rumfeld ~~will~~ would say 4 equations, 4 known unknowns)

Simplify

$[OH^-] = \frac{K_w}{[H_3O^+]}$, substitute into charge balance eqn.

$$[H_3O^+] = [A^-] + \frac{K_w}{[H_3O^+]}$$

$[HA] = [HA]_0 - [A^-]$, substitute into K_a

$$K_a = \frac{[H_3O^+][A^-]}{[HA]_0 - [A^-]}$$

$[A^-] = [H_3O^+] - \frac{K_w}{[H_3O^+]}$, substitute into K_a above

$$K_a = \frac{[H_3O^+] \left([H_3O^+] - \frac{K_w}{[H_3O^+]} \right)}{[HA]_0 - \left([H_3O^+] - \frac{K_w}{[H_3O^+]} \right)} \Rightarrow K_a = \frac{[H_3O^+]^2 - K_w}{[HA]_0 - \left(\frac{[H_3O^+]^2 - K_w}{[H_3O^+]} \right)}$$